Hybrid Fuzzy Control of the Inverted Pendulum via Vertical Forces

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In this article, we look at the excellent effect of vertical force as regards the stabilization of the inverted pendulum (IP) and demonstrate how the fuzzy control design methodology can be used to construct a hybrid fuzzy control system that incorporates PD control into a Takagi–Sugeno fuzzy control structure for stabilizing the IP via a vertical force. By gaining an intuitive understanding of the dynamics of the IP, the IP state space is fuzzily divided into six regions. In each region, a PD controller is designed to satisfy the stability conditions obtained by Lyapunov's direct and indirect methods. It shows that the proposed hybrid fuzzy control scheme provides a more flexible and intuitive way to stabilize the IP via a vertical force. © 2005 Wiley Periodicals, Inc.

1. INTRODUCTION

The inverted pendulum (IP) is one of the most popular examples used for illustrating various control techniques. This system is highly motivated by applications such as the control of rockets and the antiseismic control of buildings. The goal of controlling the IP is to balance the pendulum in the upright position when it initially starts with some nonzero angle off the vertical position. This is a very typical and academic nonlinear control problem, and many techniques already exist for its solution,[1–5] for example, model-based control, fuzzy control, neural network (NN) control, genetic algorithms (GAs)-based control, and so on. However, it is rather surprising that most existing technical literature refers to the planar pendulum with one degree of freedom. Only very recently have a few references

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dealing with the spherical pendulum with two degrees of freedom appeared.[6–8] Due to the complex control problems involved, Ref. 6 addresses the stabilization of the spherical IP by simultaneously controlling two uncoupled planar pendulums (the respective projections on the two orthogonal planes of the intertnal coordinate system). References 7 and 8 apply the method of controlled Lagrangians to get theoretical stability conditions for the spherical IP.

Conversely, some authors have considered an alternative control action consisting of an oscillatory vertical force applied to the pendulum pivot. The stabilizing effect of a fast vertical oscillation applied to the pendulum base is known from the early work of Stephenson in 1908.[9] The Russian physicist Kapitsa was the first, in the 1950s, to produce a rigorous demonstration of the stability conditions of the IP when its suspension base oscillates at a high frequency[10] and, therefore, some authors use the expression Kapitsa pendulum to refer to this stabilization technique.[11] Another control alternative is based on the application of a rotational torque to the pendulum base, as proposed by Furuta and coworkers.[12] In fact, this arrangement leads to a different kind of planar IP, known as the rotational IP[13] or, simply, the Furuta pendulum.[14]

With the exception of vibrational control, that is, control based on oscillatory control signals, which is a well-known technique for controlling mechanical systems,[15–17] including, as mentioned above, the IP,[18–20] the only previous work, to our knowledge, that considers the application of vertical forces to stabilize the IP was recently performed by Wu et al.,[21,22] who employ the IP as a basic element to analyze the postural stability and locomotion of multilink bipeds. In particular, Wu et al. model the IP base point according to cartilage and ligament behavior in natural joints and they apply horizontal and vertical forces and, also, a rotational torque to the base pivot. Using a very simplified linear model, the resulting overactuated control system is designed by means of Lyapunov's direct method to obtain a desired trajectory of the IP's center of gravity.

Controlling the pendulum with two or three degrees of freedom is still an extremely challenging problem in this area. Humans, on the other hand, seem to easily balance the pendulum, often relying on simple intuitive knowledge about the dynamics of the pendulum; for example, while the pendulum is falling over to the right-hand side, one must move his/her finger to the right-hand side at once. It is interesting to note that humans not only use horizontal but also *vertical* forces to stabilize the pendulum, for example, when the pendulum is falling, the vertical force should be immediately activated to produce a strong negative vertical acceleration to assist the pendulum to return to the vertical position. The excellent stabilization effect of the vertical force was first noted and extensively studied by Maravall,[23] who investigated the combination of the vertical force with the customary horizontal force, arriving at the stabilization conditions for different formal representations of the system and obtaining the stability conditions of the IP for a PD control algorithm.

In this article, by using *intuitive* knowledge about stabilizing the IP via vertical forces, we propose a hybrid fuzzy control system that incorporates PD control into a Takagi–Sugeno (T-S) fuzzy control structure.[24] The article is organized as follows. In Sections 2 and 3, we give a brief review of the control and stabilization of the IP via vertical force, as well as its combination with the customary horizontal force.[23] The hybrid fuzzy control algorithm is presented in Section 4. In Section 5, we discuss how the intuitive IP control knowledge can be used to tune the proposed fuzzy controller. Finally, Sections 6 and 7 discuss experimental results and give some conclusions and remarks, respectively.

2. STABILIZATION OF THE INVERTED PENDULUM WITH A VERTICAL FORCE[23]

In practice, the only control action used in the technical literature for IP stabilization is a horizontal force, which is almost universally materialized by means of an electrical cart, that is, the popular cart–pole system shown in Figure 1a. In this article, we consider the novel idea of controlling the IP system via a vertical force F_y as shown in Figure 1b.

For the IP with a vertical force, by applying Lagrange's Equations, the system dynamics can be expressed in the standard compact form as follows[23]:

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = \tau \tag{1}$$

where $M(\mathbf{q})$ is the symmetric, definite positive inertia matrix, $C(\mathbf{q}, \mathbf{q})$ is the Coriolis/centripetal matrix, and $G(\mathbf{q})$ is the gravity vector

$$\mathbf{q} = \begin{bmatrix} y \\ \theta \end{bmatrix}; \qquad M(\mathbf{q}) = \begin{bmatrix} (M+m) & -ml\sin\theta \\ -ml\sin\theta & ml^2 \end{bmatrix}$$
$$C(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} 0 & -ml\cos\theta\dot{\theta} \\ 0 & 0 \end{bmatrix}; \qquad G(\mathbf{q}) = \begin{bmatrix} (M+m)g \\ -mgl\sin\theta \end{bmatrix}$$
$$\tau = \begin{bmatrix} F_y \\ 0 \end{bmatrix} \tag{2}$$

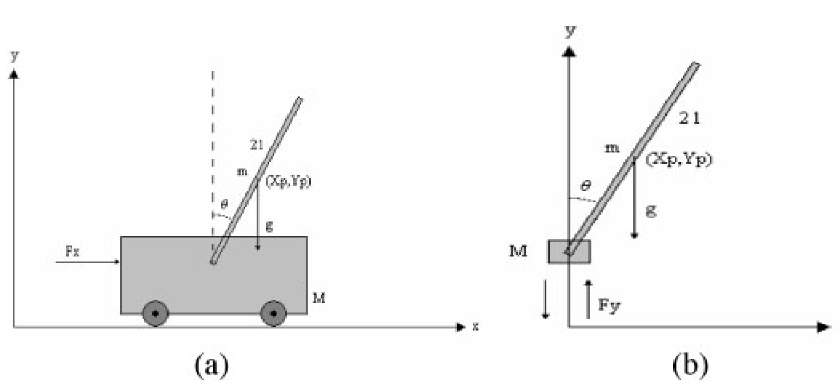

Figure 1. Planar inverted pendulum supported by a platform subject to (a) a horizontal force F_x and (b) a vertical force F_y.

As a general conclusion,[23] the application of a single vertical force to stabilize and control the IP is unfeasible, although its excellent and, in particular, its fast stabilization effect makes the combination of the vertical force with the customary horizontal force look very attractive, which is our next topic.

3. COMBINATION OF HORIZONTAL AND VERTICAL FORCES

After having introduced the stabilization of the IP by means of a vertical force, we are now going to explore its combination with the usual horizontal force. The mechanism for implementing the vertical force, F_y, is a platform of mass m', mounted on the customary electrical cart of mass M, which, as usual, produces the horizontal force, F_x.

The total kinetic energy of the cart–platform–pendulum ensemble is

$$K = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} m'(\dot{x}^2 + \dot{y}^2) + \frac{1}{2} m(\dot{x}_p^2 + \dot{y}_p^2) \tag{3}$$

where m is the mass of the pole. The potential energy is

$$P = m'gy + mgy_p \tag{4}$$

By applying Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = F_i \quad q_1 = x, \quad F_1 = F_x; \qquad q_2 = y, \quad F_2 = F_y;$$
$$q_3 = \theta, \quad F_3 = 0 \tag{5}$$

we get the global system dynamics:

$$\begin{aligned}(M + m' + m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 &= F_x \\ (m + m')\ddot{y} - ml\sin\theta\ddot{\theta} - ml\cos\theta\dot{\theta}^2 &= F_y - (m + m')g \\ \cos\theta\ddot{x} - \sin\theta\ddot{y} + l\ddot{\theta} - g\sin\theta &= 0\end{aligned} \tag{6}$$

To analyze this highly nonlinear system, let us first make the following qualitative remarks.

(1) From the study of the pure vertical force case,[23] we know that in this case the only exogenous control action is vertical acceleration, which leads to the unfeasible stabilization strategy of maintaining the platform in free fall.
(2) On the other hand, the pure horizontal force can be used as a feedback control action that straightforwardly stabilizes the IP.
(3) Due to the equivalence of the joint (x, θ) dynamics of the combined case given by Equation 6 and the pure horizontal case, we can, in principle, exploit the feedback stabilization capacity of force F_x by focusing on the joint (x, θ) dynamics of the horizontal plus the vertical case and considering vertical acceleration as an additional external control action.

Thus, let us rewrite the joint (x, θ) dynamics of the combined case:

$$(M + m' + m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 = F_x$$
$$\cos\theta\ddot{x} + l\ddot{\theta} - (g + \ddot{y})\sin\theta = 0 \tag{7}$$

which is equivalent to the pure horizontal case,[23] except that the gravity dynamics is perturbed by the term $\ddot{y}\sin\theta$. Remember that this perturbation is generated by the vertical dynamics, given by the second equation of Equation 6.

Thus, by considering vertical acceleration as an exogenous element in the pendulum dynamics, the combined forces case turns out to have the same formal structure as the horizontal force case. Therefore, we can tackle the combined case as an ordinary differential equation (ODE) problem and stabilize the IP via the horizontal force F_x using any standard control law, as in Ref. 23. Alternatively, we can also approach the control problem with the state variable representation and stabilize the IP with a plethora of available techniques, including Lyapunov's direct and indirect methods. However, let us concentrate our study on the ODE approach, which conveys a very intuitive and direct physical interpretation.

3.1. Ordinary Differential Equations Analysis

As usual, we are interested in the neighborhood of the IP vertical position, so that we linearize the system dynamics Equation 7 by approximating $\cos\theta \approx 1 - \theta^2/2$, $\sin\theta \approx \theta$, $\theta^2 \ll 0$. After solving the ODE system in θ, we get

$$\ddot{\theta} - \frac{M + m' + m}{Ml}(g + \ddot{y})\theta = -\frac{F_x}{Ml} \tag{8}$$

which is virtually equivalent to the pure horizontal case

$$\ddot{\theta} - \frac{M + m}{Ml} g\theta = -\frac{F_x}{Ml} \tag{9}$$

Thus, let us apply a PD control law of the error variable

$$F_x = -k_p e - k_d \dot{e} = k_p\theta + k_d\dot{\theta} \tag{10}$$

which, substituted into Equation 9, yields

$$\ddot{\theta} + \frac{k_d}{Ml}\dot{\theta} + \left[\frac{k_p}{Ml} - \frac{M + m' + m}{Ml}(g + \ddot{y})\right]\theta = 0 \tag{11}$$

Again, it is equivalent to the closed-loop horizontal force case, whose ODE[23] is

$$\ddot{\theta} + \frac{k_d}{Ml}\dot{\theta} + \left[\frac{k_p}{Ml} - \frac{M + m}{Ml} g\right]\theta = 0 \tag{12}$$

the only remarkable effect of the vertical acceleration being on the root locus of the combined forces case. As is well known, both coefficients must be positive to guarantee stability of Equation 11:

$$\frac{k_d}{Ml} > 0 \rightarrow k_d > 0; \qquad k_p > (M + m' + m(g + \ddot{y})) \tag{13}$$

The first condition has a straightforward interpretation; namely, it implies that the feedback control force, F_x, must have a component directly proportional to the pendulum angular speed θ. To illustrate this fact, the four possible states of the IP have been represented in Figure 2. Note that in cases b and d the pendulum is returning to its vertical position, whereas in cases a and c it is moving away from it. In all cases, the orientation of the corresponding control force has been indicated.

Note that the vertical force-based IP stabilization can be explained in an intuitive way. When the pendulum is falling, the vertical acceleration $\ddot{y}$ strengthens IP stabilization. Inversely, when the platform is ascending to recover its original position, the respective positive vertical acceleration detracts from IP stabilization. Therefore, the vertical force component, that is, the generation of vertical acceleration, must be carefully designed to tackle this double-sided effect. Roughly speaking, when the pendulum is moving away from the vertical position, the vertical force should be immediately activated to produce a strong negative vertical acceleration. Conversely, with the pendulum recovering its vertical position, we can make $F_y > 0$ to bring the platform toward its original position. In short, the vertical displacement of the platform must be synchronized with the IP movements. This general control strategy can be succintly formalized as follows[23]:

$$\textit{If}\ [\mathrm{sgn}(\theta) = \mathrm{sgn}(\dot{\theta})]\ \textit{then}\ [F_y > 0 \rightarrow \ddot{y} < 0]\ \textit{else}\ [F_y > 0 \rightarrow \ddot{y} > 0] \tag{14}$$

Apart from controlling the IP's deviation angle, which is obviously the main goal, it is also of interest to minimize the platform displacement, which must be constrained to some specific range. To this end, let us distinguish the following three states of the cart–platform–pendulum ensemble.

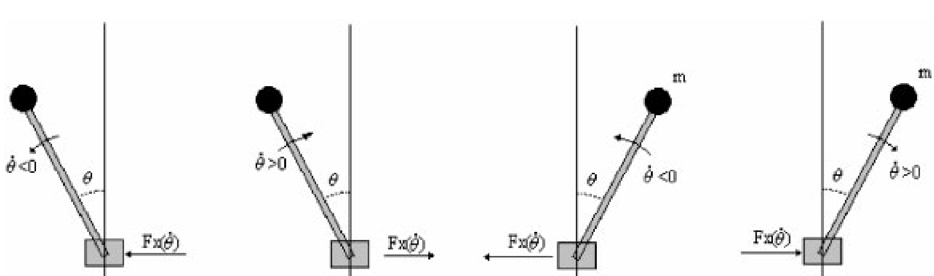

Figure 2. The four possible IP states. Observe the orientation of the respective feedback force.

(1) The IP is moving away from the vertical position, that is, $\mathrm{sgn}(\theta) = \mathrm{sgn}(\dot{\theta})$.
(2) The IP is returning to the vertical position, that is, $\mathrm{sgn}(\theta) \neq \mathrm{sgn}(\dot{\theta})$, but is still *far* from it.
(3) As in state 2, but *near* the vertical position.

Accordingly, the following control action for each IP state can be derived.[9]

(1) The pendulum is leaving the vertical position:

$$F_y(t) = -[k_{p\theta}|\theta(t)| + k_{d\theta}|\dot{\theta}(t)|] \tag{15}$$

(2) The pendulum is returning to and is *far* from the vertical position:

$$F_y(t) = -p[k_{p\theta}|\theta(t)| + k_{d\theta}|\dot{\theta}(t)|] - (1-p)[k_{py}\,y(t) + k_{dy}\,\dot{y}(t)] \tag{16}$$

where $0 < p < 1$ weights the importance of the two control objectives: the IP deviation angle, θ, and the platform movement, y. Note that the latter action is aimed at minimizing the vertical displacement.

(3) The pendulum is returning to and is *near* to the vertical position:

$$F_y(t) = -[k_{py}\,y(t) + k_{dy}\,\dot{y}(t)] \tag{17}$$

The stabilization conditions for different formal representations of the IP system controller by the PD controllers, Equations 15 and 17, were given in Ref. 23. As we said before, having noted that the vertical force causes the platform of the IP to be in a free fall, Maravall[23] proposed a control strategy that combines the customary horizontal force with the vertical force. Roughly speaking, the horizontal force permits a direct stabilization of the IP by means of a feedback control action, whereas the vertical force significantly improves the IP stabilization, mainly due to its fast response to external perturbations of the IP equilibrium state.

4. FUZZY CONTROL OF THE INVERTED PENDULUM VIA A VERTICAL FORCE

In this section, we look at how intuitive knowledge can be employed to construct a fuzzy control system to stabilize the IP via a vertical force. Note that for the control strategy, Equations 15 to 17, different IP states were used for control implementation. (i) $\mathrm{sgn}(\theta) = \mathrm{sgn}(\dot{\theta})$ means the IP is moving away from the vertical position (VP), so a strong control action is needed; (ii) $\mathrm{sgn}(\theta) \neq \mathrm{sgn}(\dot{\theta})$ means the IP is returning to the VP, so different control strategies should be employed for different positions of the pendulum, for example, the pendulum is *near* or *far* from the VP. It can be seen that the fuzzy linguistic qualifiers have been used to describe the IP states. In the following, we will study how to quantify intuitive knowledge about how to control the IP using linguistic description. It is interesting to note that the condition part of the control strategy, Equation 14, only deals with the sign of θ and with the sign of $\dot{\theta}$. This exactly reflects the human control heuristics.

Based on the three states of the IP system presented in Section 2, we propose a fuzzy partition in $(\theta, \dot{\theta})$ state space of the IP as shown in Figure 3. By using thresholds a, b, c, and d, the $(\theta, \dot{\theta})$ state space can be fuzzily divided into six

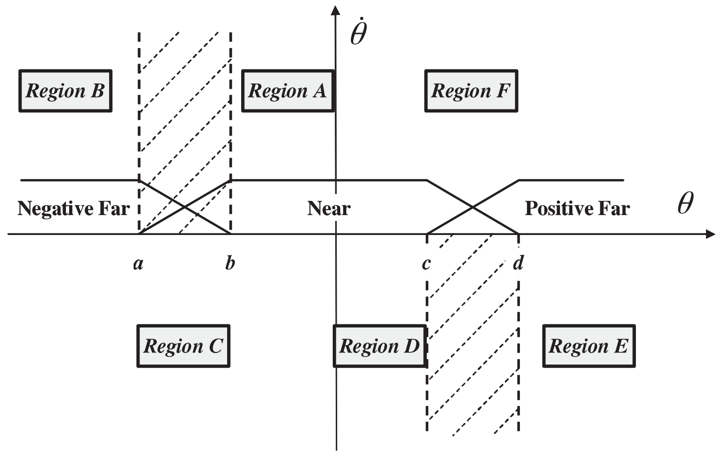

Figure 3. Fuzzy partition in $(\theta, \dot{\theta})$ state space of the IP.

regions (regions A to F). Working with these thresholds means the proposed hybrid fuzzy controller is made using six sets of fuzzy rules, that is, there is a fuzzy rule that corresponds to each of the six regions. The intuitive understanding of each region is given in Table I. By observing Figure 3 and Table I, we can see that region D is a mirror image of region A, and region E is a mirror image of region B.

Table I. Intuitive control knowledge on stabilization of the IP with a vertical force.

Region	θ	$\dot{\theta}$	IP states	Control perceptions
A	Negative near	Positive	Returning to the VP, and being *near* to the equilibrium point.	Feedback of y and $\dot{y}$ should be employed to restrict the falling of the IP.
B	Negative far	Positive	Returning to the VP, but being still *far* from the equilibrium point.	A strong vertical force, which depends on $\lvert\theta\rvert$ and $\lvert\dot{\theta}\rvert$, is needed to bring the IP back to the VP.
C	Negative	Negative	Leaving the VP.	A strong vertical force is needed to bring the IP back to the VP.
D	Positive near	Negative	Returning to the VP, and being *near* to the equilibrium point.	Feedback of y and $\dot{y}$ should be employed to restrict the falling of the IP.
E	Positive far	Negative	Returning to the VP, but being still *far* from the equilibrium point.	A strong vertical force, which depends on $\lvert\theta\rvert$ and $\lvert\dot{\theta}\rvert$, is needed to bring the IP back to the VP.
F	Positive	Positive	Leaving the VP.	A strong vertical force is needed to bring the IP back to the VP.

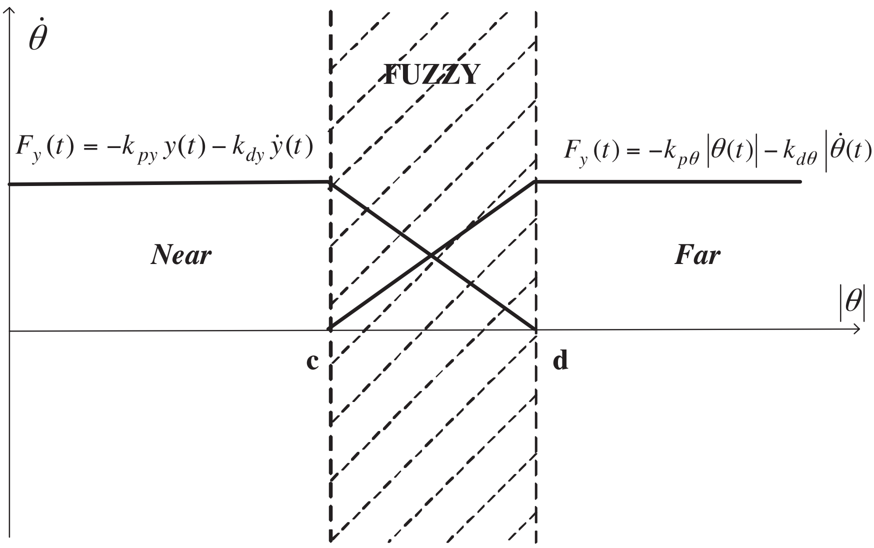

Figure 4. Fuzzy partition in $(|\theta|, \dot{\theta})$ state space of the IP $(\mathrm{sgn}(\theta) \neq \mathrm{sgn}(\dot{\theta}))$.

So, the fuzzy partition can be further interpreted in a compact way as shown in Figure 4.

Based on the intuitive understanding of controlling the IP with a vertical force and the control strategy Equations 15 to 17, we propose the following hybrid fuzzy control that incorporates PD control into a Takagi–Sugeno (T-S) fuzzy control structure.[24]

IF $\mathrm{sgn}(\theta) = \mathrm{sgn}(\dot{\theta})$, THEN $F_y(t) = -[k_{p\theta}|\theta(t)| + k_{d\theta}|\dot{\theta}(t)|]$

ELSE

IF $|\theta|$ is *Far*, THEN $F_y(t) = F_1 = -[k_{p\theta}|\theta(t)| + k_{d\theta}|\dot{\theta}(t)|]$

IF $|\theta|$ is *Near*, THEN $F_y(t) = F_2 = -[k_{py}\, y(t) + k_{dy}\, \dot{y}(t)]$

By using T-S inference,

$$F_y = \frac{\sum_{i=1}^{2} w_i F_i}{\sum_{i=1}^{2} w_i} = \sum_{i=1}^{2} \overline{w}_i F_i \tag{18}$$

where $\overline{w}_i = w_i / \sum_{i=1}^{2} w_i$, $w_1 = \mu_{Far}(|\theta|)$, and $w_2 = \mu_{Near}(|\theta|)$. The design procedure for PD controllers F_1 and F_2 is given in Ref. 23.

Note $\sum_{i=1}^{2} \overline{w}_i = 1$, so the T-S fuzzy controller Equation 7 can be rewritten as

$$\begin{aligned} F_y &= \overline{w}_1 F_1 + \overline{w}_2 F_2 = \overline{w}_1 F_1 + (1 - \overline{w}_1) F_2 \\ &= -\overline{w}_1 [k_{p\theta} |\theta(t)| + k_{d\theta} |\dot{\theta}(t)|] - (1 - \overline{w}_1)[k_{py} y(t) + k_{dy} \dot{y}(t)] \end{aligned} \tag{19}$$

Comparing the above fuzzy controller with the control strategy, Equation 16, we can see that both have the same control structure but in Equation 19 the weight factor for the two PD controllers, Equations 15 and 17,

$$\overline{w}_1 = \frac{w_1}{\sum_{i=1}^{2} w_i} = \frac{\mu_{Far}(|\theta|)}{\mu_{Far}(|\theta|) + \mu_{Near}(|\theta|)}$$

can be computed according to the fuzzy inference method whereas the weight factor p in Equation 16 is subjectively selected. Please also note that $\overline{w}_1$ changes along with the pendulum deviation θ, but p is fixed.

5. DISCUSSION

In this section, we discuss how the intuitive IP control knowledge can be used to tune the hybrid fuzzy controller. Note that the T-S fuzzy controller, Equation 7, does not include the falling state, where $\mathrm{sgn}(\theta) = \mathrm{sgn}(\dot{\theta})$. Therefore, we only focus on the returning state as shown in Figure 3, where the fuzzy membership functions for describing *Far* and *Near* can be defined as follows:

$$w_1 = \mu_{Far}(|\theta|) = \begin{cases} 0 & \theta \leq c \\ \dfrac{|\theta| - c}{d - c} & c < \theta \leq d \\ 1 & \theta > d \end{cases}$$

$$w_2 = \mu_{Near}(|\theta|) = \begin{cases} 1 & \theta \leq c \\ \dfrac{d - |\theta|}{d - c} & c < \theta \leq d \\ 0 & \theta > d \end{cases} \tag{20}$$

From Equation 20, we have $w_1 + w_2 = 1$, $\overline{w}_1 = \mu_{Far}(|\theta|)$ and $\overline{w}_2 = \mu_{Near}(|\theta|)$. Hence, the T-S fuzzy controller Equation 19 can be rewritten as

$$\begin{aligned} F_y &= \overline{w}_1 F_1 + \overline{w}_2 F_2 = \mu_{Far}(|\theta|) F_1 + \mu_{Near}(|\theta|) F_2 \\ &= \begin{cases} F_2, & |\theta| \leq c \\ \dfrac{|\theta| - c}{d - c} F_1 + \dfrac{d - |\theta|}{d - c} F_2, & c < |\theta| \leq d \\ F_1, & |\theta| > d \end{cases} \end{aligned} \tag{21}$$

Table II. Effect of the fuzzy partition (upper boundary d).

d	y	θ	Remarks
Increasing	Stronger control	Weaker control	We take a stronger effect on the goal of returning the platform to the original position. This implies that we care more about y but less about θ.
Decreasing	Weaker control	Stronger control	We employ a stronger effect on the goal of returning the IP to the vertical position. This implies that we care more about θ but less about y.

It can be seen that the PD controller F_1 is valid on a region of the state space that is quantified via $\overline{w}_1$ and another PD controller F_2 is valid on the region quantified via $\overline{w}_2$ (with a fuzzy boundary in between as shown in Figure 4). We may conclude that the T-S fuzzy system provides a very intuitive representation of the IP controller as a *nonlinear* interpolation between the two *linear* controllers F_1 and F_2. Considering that the PD controller F_1 is used to control pendulum angle θ whereas the PD controller F_2 has control effect on y to bring the platform back to its original position, the effect of the upper boundary of the fuzzy partition can be outlined as shown in Table II.

In a similar way, we can intuitively describe the effect of the lower boundary c of the fuzzy partition, as outlined in Table III.

Then, we can conclude that the lower boundary c is of secondary importance in the tuning of the hybrid fuzzy control, as compared with the strong influence of the upper boundary d on the IP stabilization.

6. EXPERIMENTAL RESULTS

We very briefly present some simulation results obtained by combining horizontal and vertical forces and by applying the hybrid fuzzy control. We have considered the values of the system parameters to be as follows: cart mass, 2 kg; platform mass, 0.2 kg; pendulum mass and length, 0.1 kg and 0.5 m, respectively. Unless otherwise indicated, distances are in meters, time in seconds, forces in

Table III. Effect of the fuzzy partition (lower boundary c).

c	y	θ	Remarks
Increasing	Slightly stronger control	Slightly weaker control	We have a very slightly stronger effect on the vertical displacement y. θ has a slightly weaker control.
Decreasing	Slightly weaker control	Slightly stronger control	We have a very slightly stronger effect on angular deviation θ. The vertical displacement y has a slightly weaker control.

newtons, and angular displacements in radians in all figures. In all the reported examples the gains of the PD algorithm have been obtained to optimize the usual performance indices: rise time, overshoot peak, and settling time. To get an idea of how much improvement in the IP stabilization can be achieved with the addition of a vertical force, we have also included the performance of the pure horizontal force.

As regards the effect of the fuzzy parameters c and d on the IP stabilization, we have run simulations for a variety of values, confirming the discussion outlined in Tables II and III. Figure 5 shows the IP trajectories for different values of the lower boundary $c = 1°$, $4°$, and $8°$. In all these cases the initial IP deviation is 25°. As a general conclusion, the lower the parameter c is, the stronger is the recuperation of the IP. As a logical consequence, the corresponding control efforts are inversely proportional to the magnitude of the lower boundary c. However, as shown in Figure 5, the differences in performance and control efforts due to the values of c are not significant. A balanced choice could be $c = 4°$, as the IP trajectory is quite good, and the corresponding control efforts, in particular the vertical displacement, are very acceptable.

As regards the upper boundary d, Figure 6 shows the IP trajectory, the horizontal and the vertical motions, and the profile of the vertical force for the same initial deviation of 25° and for $d = 10°$, $15°$, and $20°$. Curiously enough, the lower the value of d, the stronger the recuperation of the IP. Logically, the corresponding control efforts are inversely proportional to the magnitude of the upper boundary. Unlike the lower boundary case, the trajectories of the IP and the control efforts for different values of the upper boundary present more significant differences. So, as regards the IP trajectory, we can observe that the IP reaches the equilibrium position for the first time at approximately 0.2 s for $d = 10°$ and at 0.3 s for $d = 20°$. Note, also, the significant difference in the vertical displacements for these values of the upper boundary (45 cm for $d = 10°$ and 35 cm for $d = 10°$). Concerning the magnitude of the vertical force, for $d = 10°$ the maximum force is about 30 N and 20 N for $d = 20°$. Hence, a compromise between the IP recuperation and the corresponding control efforts could be to choose the upper limit within the range 15°–20°.

Finally, we present in Figure 7 a comparative study of different alternatives to stabilizing the IP. To get an idea of how much improvement in the IP stabilization can be achieved with the addition of a vertical force, we have included the performance of the customary horizontal force. The *crisp* combination of horizontal and vertical forces (see Equation 15, with $p = 0.6$) has also been shown. We can observe in Figure 7 that the best stabilization is obtained with the hybrid fuzzy control for $c = 4°$ and $d = 10°$, although the corresponding maximum vertical displacement is approximately 45 cm and the vertical force reaches a peak of 30 N. The same hybrid fuzzy control for $c = 4°$ and $d = 20°$ gives a quite similar IP stabilization, but at a lower control effort, so that it could be the optimum choice.

7. CONCLUDING REMARKS

In this article, based on the novel idea of controlling and stabilizing the IP via vertical forces proposed by Maravall,[23] we have demonstrated how the fuzzy con-

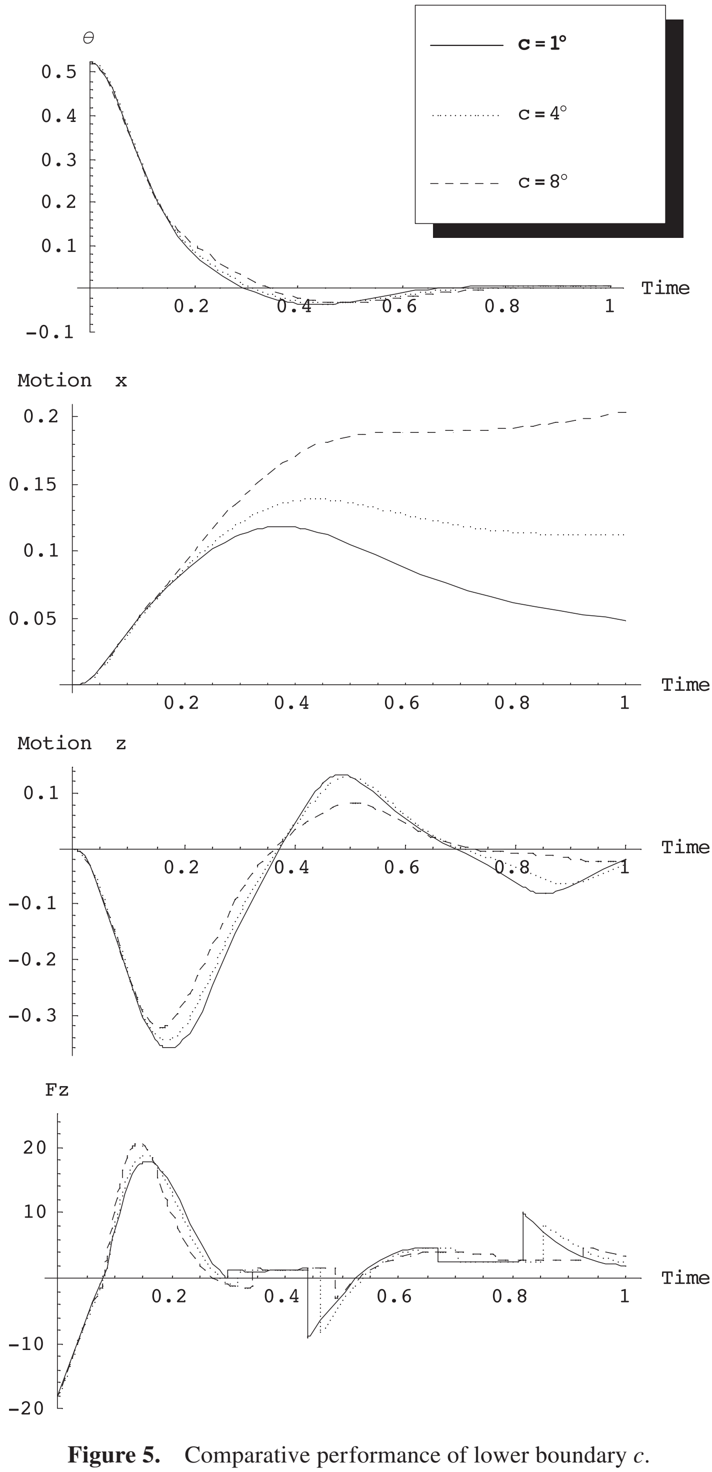

Figure 5. Comparative performance of lower boundary *c*.

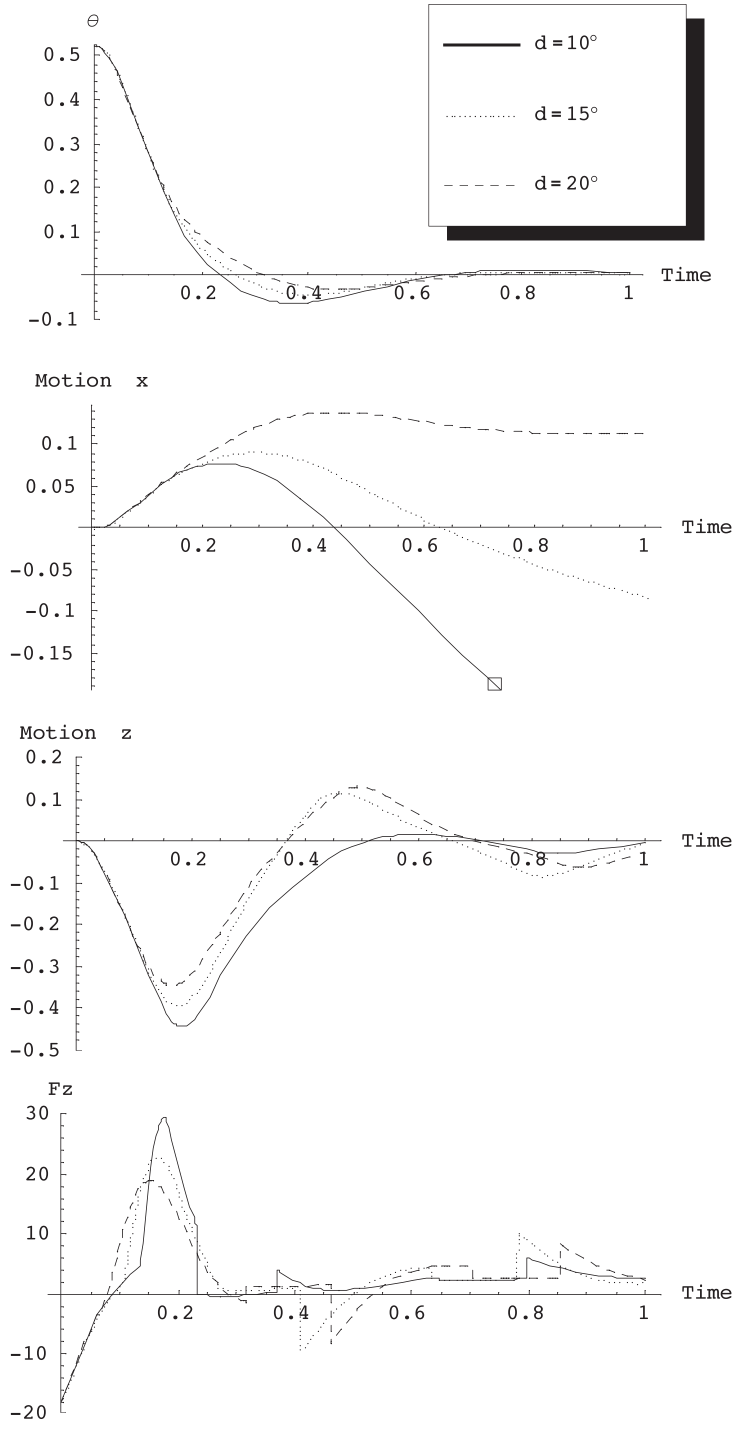

Figure 6. Comparative performance of upper boundary *d*.

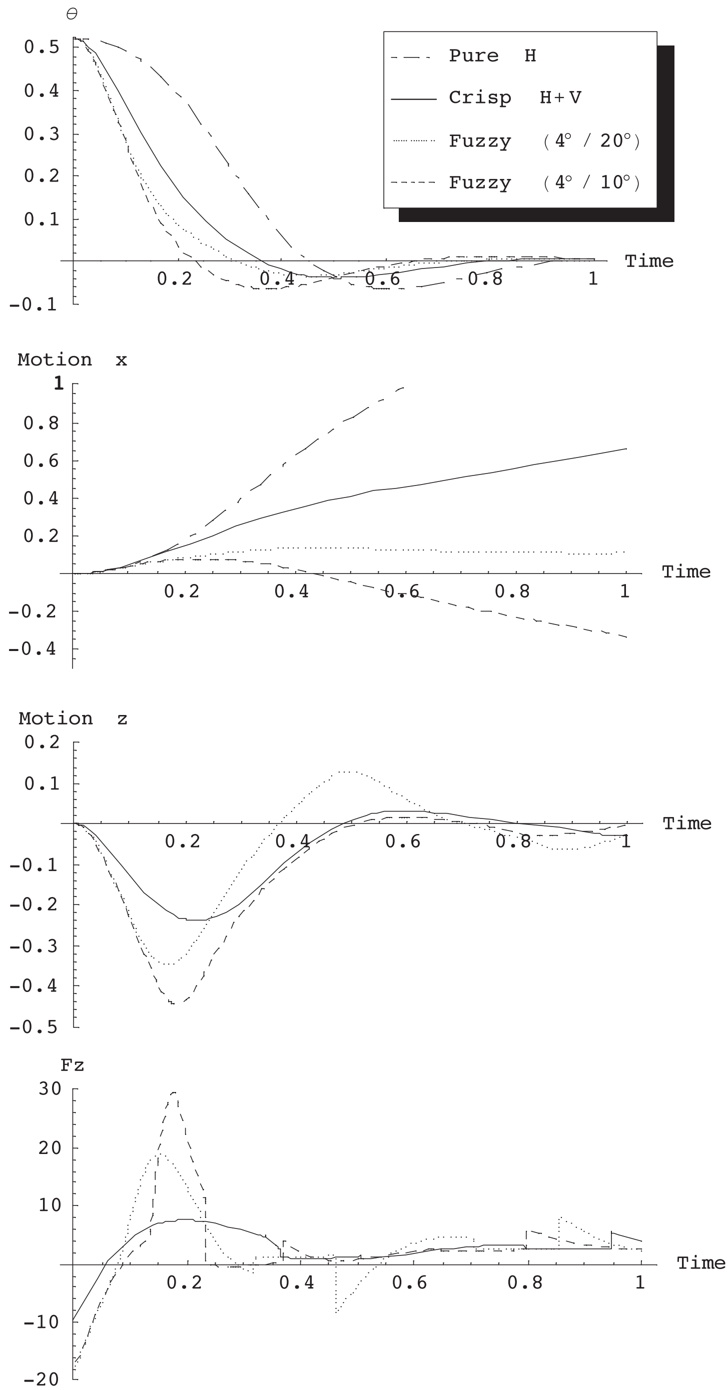

Figure 7. Comparative performances of pure horizontal control, crisp horizontal plus vertical control, and hybrid fuzzy control.

trol design methodology can be used to construct a hybrid fuzzy control system, which incorporates PD control into a T-S fuzzy control structure, to stabilize the vertical force-based IP in a more flexible and intuitive way.

How to design a trainable hybrid fuzzy controller for stabilization of the IP via both vertical and horizontal forces is a new challenge in this field. To further improve the performance of the IP stabilization, learning is a naturally good choice, for example, learning to partition the IP state space and so on. We are currently working on learning to stabilize the IP via both vertical and horizontal forces using soft computing techniques.

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