Controller for Urban Intersections Based on Hybrid Automaton

S. Hassan HosseinNia and Blas M. Vinagre
Dept. of Electrical, Electronic and Automation Engineering,
Industrial Engineering School,
University of Extremadura,
Badajoz, Spain
Email: shossein@alumnos.unex.es; bvinagre@unex.es

Vicente Milanés and Carlos González
Industrial Computer Science Department,
Instituto de Automática Industrial,
Consejo Superior de Investigaciones Científicas,
Madrid, Spain
Email: vmilanes,gonzalez@iai.csic.es

***Abstract*— In this paper, the problem of crossing the intersections is studied. A controller is designed based on a hybrid automaton. The design procedure is presented into two cases, problem crossing one automatic car and one manual car from intersection is studied in case one. And following of two automatic cars apart from their safe crossing (i.e. fixing safe distance between two automatic car), are studied case two. Simulation is done by the model which is identified by the real data from the car Citroen C3. Simulation results show the efficiency of the controller in both cases.**

I. Introduction

Future traffic systems could include vehicles that are automatically driven along smart highways and/or city roads [1], [2]. Such automation of traffic movement, along with a control infrastructure for the overall coordination and scheduling of traffic, could improve efficiency, reduce travel time, and prevent traffic deadlocks. Crossroad are one of the major problems in controlling the flow of vehicular traffic in urban environments. Approximately 50% of all traffic collisions occur at crossroads, and over 60% are in urban areas [3]. There are two principal issues affecting the safety of traffic at crossroads. One is poor visibility of the minor approaches to a crossroad so that drivers may fail to yield when they should. The other is inadequate traffic signaling in areas with a continuing series of crossroads [4]. These two problems may be a cause of vehicle collisions related to the "dilemma zone," i.e., the space from the crossroad to the point on the road where it is difficult for the driver to discern whether he should accelerate, thus passing the crossroad before the other vehicle, or decelerate to play it safe [4].

Many works has been reported during last two decayed. Febbraro et.al., has been worked on modeling and control of urban transportation networks using hybrid Petri nets [5],[6],[7]. Hybrid system design for formations of autonomous vehicles is proposed in [8]. In [9], the authors considered the problem of scheduling automated traffic in a city which can improve efficiency of the system by decreasing delays, increasing capacity, and easing congestion. Cooperative driving technology with intervehicle communication is proposed to improve driving safety and efficiency using appropriate motion scheduling of all the encountered vehicles in [10].

Recently, adaptive cruise control with Stop&Go maneuvers and control of urban intersections using Fuzzy control are presented in [11],[4]. In this work, adaptive cruise control with Stop&Go maneuvers and control of urban intersections is proposed using the hybrid techniques.

The rest of this paper organized as follows. Hybrid system, and automaton will be briefly described in section II. Our problem will be illustrated into two cases in section III, and finally this paper will be concluded in section IV.

II. Hybrid Systems

The term hybrid systems is used in the literature to refer to systems that feature an interaction between diverse types of dynamics. Most heavily studied in recent years are hybrid systems that involve the interaction between continuous dynamics and discrete dynamics. Many physical systems today are modelled by interacting continuous and discrete event systems . Such hybrid systems contain both continuous and discrete states that influence the dynamic behaviour . There is a lot of interest in these kinds of systems today , since a large number of systems are neither pure continuous nor discrete but a combination . This is mostly due to the growing use of computers in the control of physical plants but also as a result of the hybrid nature of many physical processes .Such systems are common across a diverse range of application areas. Examples include power systems [12], robotics [13],[14], manufacturing [15] and air traffic control [16]. The study of this class of systems has to a large extent been motivated by applications to embedded systems and control.

Control problems have been at the forefront of hybrid systems research from the very beginning. The reason is that many important applications with prominent hybrid dynamics come from the area of embedded control. For example, hybrid control has played an important role in applications to avionics, automated highways, auto motive control, air traffic management, industrial process control, and manufacturing and robotics.

The dynamics of hybrid systems will be summarize with a model called a hybrid automaton. A hybrid automaton is a mathematical model for precisely describing systems where computational processes interact with physical processes. Its behavior consists of discrete state transitions and continuous evolution. Hybrid automaton is a collection,

$$H = (Q,\ X,\ f,\ D,\ E, G,\ R) \tag{1}$$

where,

- $q \in Q$ is discrete state,
- $X \in \mathbb{R}^n$ is continuous state,
- $Init \subseteq Q \times \mathbb{R}^n$ is initial state,
- $f : Q \times X \to TX$ is a vector field,
- $D : Q \to P(X)$ is Domain,
- $E \subseteq Q \times Q$ is a set of edges,
- $G : E \to P(X)$ is a guard condition,
- $R : E \times X \to P(X)$ is a reset map.

In the following, a hybrid controller will be presented using hybrid automaton.

III. Our problem

In this section, two cases are considered to model a safe crossing. In the first case it is assumed that there are one automatic car and one manual car. The aim in this section is to design a hybrid controller in order to have a safe crossing. In the next case, two automatic cars and one manual car are considered. In this is case, the automatic cars should have safe crossing the same as previous case. Moreover, the second automatic car should follow the first automatic car. In the next section the kinematic and dynamical model of the car are described.

A. Model of the car

The modeled cars in this work are nonlinear single-track models, commonly called bicycle model [17], upgraded by steering wheel/wheel and longitudinal dynamics. Nonlinear dynamics of the vehicle is described as:

$$\begin{aligned} \dot{x} &= v_t \cos(\varphi) \\ \dot{y} &= v_t \sin(\varphi) \\ \dot{\varphi} &= v_t \tfrac{1}{L} \end{aligned} \tag{2}$$

The longitudinal dynamics avoid the instant change in the car speed. It is modeled by (3), based on the real data which is carried out from the car Citroen C3 in "Instituto de Automática Industrial (IAI), Consejo Superior de Investigaciones Científicas, Madrid, Spain". The experimental response of the dynamical system based on the reference velocity is shown in Fig. 1. This system can be modeled in many cases such as a second order dynamic system, and can therefore be described by the following differential equation:

$$\frac{v_t}{v_d} = \frac{K\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \tag{3}$$

where v_t, v_d are velocity and reference velocity, respectively. Based on the real data the unknown parameters are identified as, $K = 0.99, \varsigma = 0.69,\ \omega_n = 1.7$.

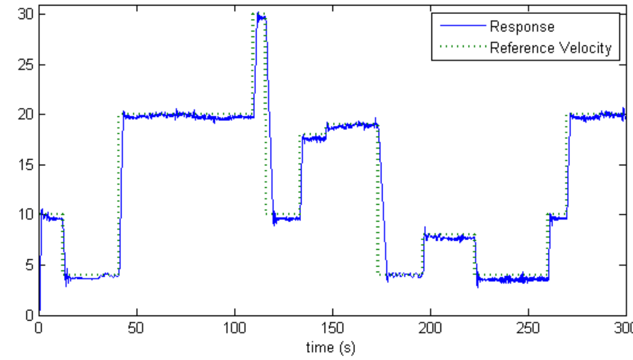

Fig. 1. Experimental response for vlocity of the car Citroen C3.

The identified model and the experimental data are compared in Fig. 2. As it can be seen the identified model can be acceptable for the simulation.

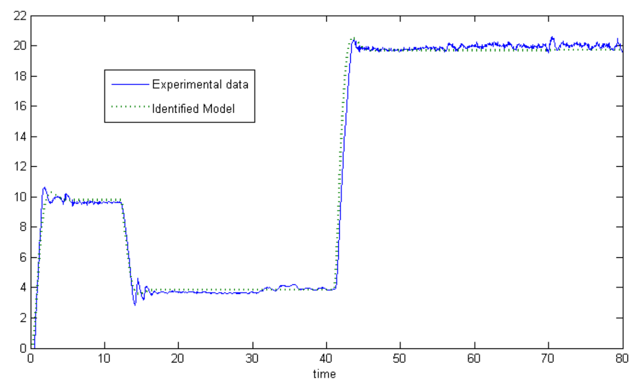

Fig. 2. Comparison between indentified model and real data.

The expanded model of the car is shown in Fig. 3.

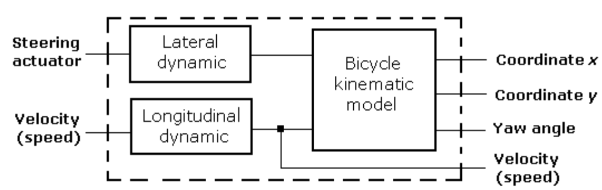

Fig. 3. Expanded model of the car

B. Case one: One automatic car and one manual car

Two checks must be made: whether a vehicle is driving along the perpendicular road and whether it is entering the crossroad. three unit vectors are defined: the autonomous vehicle's direction vector, the incoming manually driven vehicle's direction vector, and the common vector defined as the unit vector from the position of the autonomous vehicle to the position of the manual one (see Fig. 4).Therefor,three conditions have to be met to activate the crossroad controller geometrically illustrated in Fig. 4 [4].

1) The absolute value of the dot product between the autonomous vehicle's unit direction vector (d_1) and the manual vehicle's unit direction vector (d_2) must be less than 0.707, i.e., the manual vehicle is traveling along the perpendicular road.
2) The cross product of the autonomous unit direction vector (d_1) with the common unit direction vector (d_{12}) must be positive, i.e., the manual vehicle is in the right-hand segment of the perpendicular road.
3) The cross product of the autonomous unit direction vector (d_1) with the manual unit direction vector (d_2) must be negative, i.e., the manual vehicle in the right-hand segment of the perpendicular road is approaching the crossroad.

The controller will also be activated when the cars are in the radios of 30 meters of intersection point.

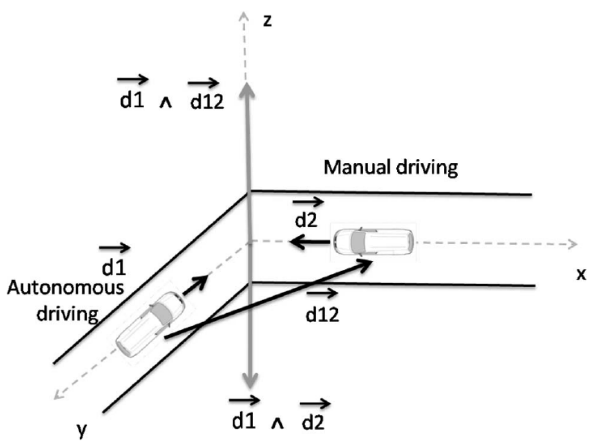

Fig. 4. Vector representation in the left-hand coordinate system.

1) Controller design: The controller is designed based on the hybrid strategy. The dynamics of hybrid systems in this case can be summarized with a hybrid automaton (as illustrated in section II), which contains the following entities :

- $Q = \{q_0, q_1, q_2\}$,
- $X = (x, y, \varphi) \in \mathbb{R}^3$,
- $Init = \{q_0\} \times \{x \in \mathbb{R}^3 : t_A \leq (t_M - 2) |\, t_A \geq (t_M + 2) \geq 0\}$,
- $f = [\ v_t \cos(\varphi), \quad v_t \sin(\varphi), \quad v_t \frac{1}{L}\ ]$, where x and y are the coordinates, φ is the vehicle orientation, v_t is the velocity of guidance point and L is the distance between front and rear axles.
- $D(q_0) = \{X \in \mathbb{R}^3 : t_A \leq (t_M - 2) |\, t_A \geq (t_M + 2) \geq 0\}$,
- $D(q_1) = \{X \in \mathbb{R}^3 :\ (t_M + 2) \leq t_A \leq (t_M + 2)\ \&\ x_A^i < 30\}$,
- $D(q_2) = \{X \in \mathbb{R}^3 :\ (t_M + 2) \leq t_A \leq (t_M + 2)\ \&\ x_A^i < 5\ \&\ x_M^i < 5\}$,
- $E = \{(q_0, q_1), (q_1, q_0), (q_0, q_2), (q_2, q_0), (q_1, q_2)\}$,

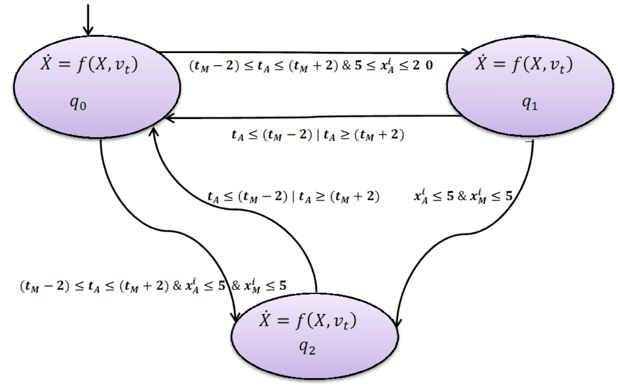

Fig. 5. Hybrid automaton: crossing safely

- $G(q_0, q_1) = \{X \in \mathbb{R}^3 :\ (t_M + 2) \leq t_A \leq (t_M + 2)\ \&\ x_A^i < 30\}$,
- $G(q_0, q_2) = \{X \in \mathbb{R}^3 :\ (t_M + 2) \leq t_A \leq (t_M + 2)\ \&\ x_A^i < 5\ \&\ x_M^i < 5\}$,
- $G(q_1, q_0) = G(q_2, q_0) = \{X \in \mathbb{R}^3 : t_A \leq (t_M - 2) |\, t_A \geq (t_M + 2) \geq 0\}$,
- $G(q_1, q_2) = \{X \in \mathbb{R}^3 :\ x_A^i < 5\ \&\ x_M^i < 5\}$,
- $R(q_0, q_1, X) = R(q_1, q_0, X) = R(q_0, q_2, X) = R(q_2, q_0, X) = R(q_1, q_2, X) = \{X\}$,

where, X is the continuous state and Q is the discrete state in the hybrid automaton. The essence of the decision making system can be explained with Fig. 5. This module is an automaton where each state is a desired car behavior. The transitions between states depend on processed information of the position and speed of the cars involved in the crossing. Position and speed of the automatic car are used to compute the time needed to cross the intersection where $t_A = \frac{x_A^i}{v_A}$, $t_M = \frac{x_M^i}{v_M}$ denote the arrival time to intersection i, and x_A^i , x_M^i denote the distance to intersection i, and v_A, v_M denote the velocity of the automatic car and manual car, respectively.

As it is mentioned in Hybrid automaton, there are three discrete states $Q = \{q_0, q_1, q_2\}$. q_0 is the initial state of the hybrid automaton which it is meant the car is in its normal speed and follows the reference speed. q_1 is the state which the car starts to reduce its speed and q_2 is the state that is there is a risk of collision and the car will be reduce its speed strongly and eventually it stop. Domain of each state is shown in Figs. 6 and 7. domain of q_0 can be easily determined by Fig. 6 but domain of q_1 and q_2 will be determined by Figs. 6 and 7, simultaneously.

Finally, discrete reference input in each state will be achieved as :

$$v_d(t+1) = \begin{cases} V_{\max}, \text{ if } Q = q_0 \\ 0.9 v_d(t), \text{ if } Q = q_1 \\ 0.5 v_d(t), \text{ if } Q = q_2 \end{cases} \tag{4}$$

where $V_{\max}$ is the maximum value for the velocity and v_t will be achieved based on longitudinal dynamics i.e. Eq. 3.

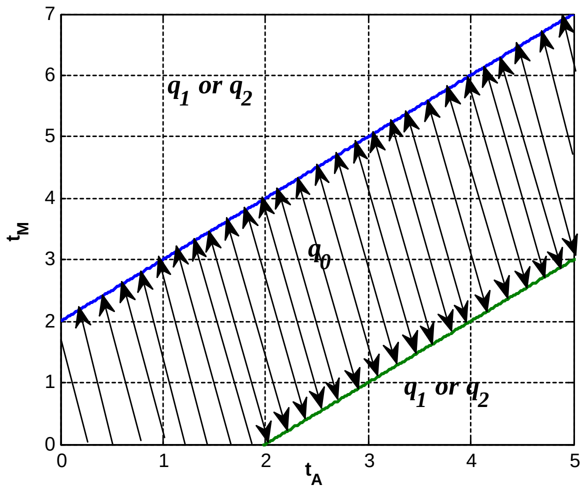

Fig. 6. Domain of q_0 and $\{q_1, q_2\}$

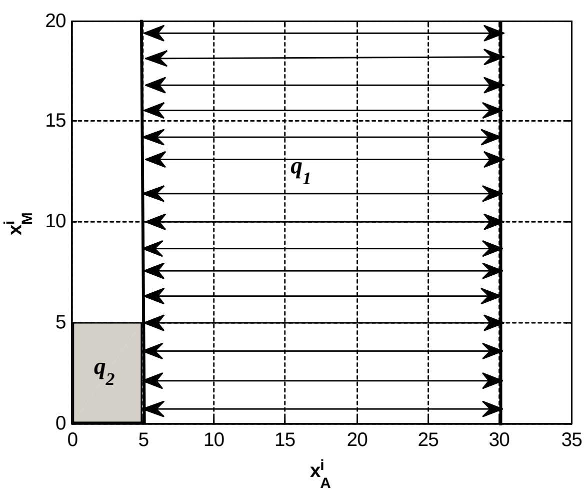

Fig. 7. Domain of q_1 and q_2

2) Simulation result: Simulation has been carried out by SIMULINK using stateflow. Maximum velocity for automatic cars are chosen as 30 Km/h . The simulation result is shown in Fig. 8. The top figure shows the variation of the velocity of each car during the crossing and the down one shows the distance between each car to intersection, where the zero value of this distance means the car is going to cross the intersection. In this case the maximum velocity and distance between automatic car and manual car to intersection are chosen as $(30\ Km/h,\ 95\ m)$ and $(25\ Km/h,\ 80\ m)$, respectively. As it can be seen from Fig. 8, there is risk of collision and the decision block is decided to reduce the speed of automatic car at $t \approx 12s$ and the cars are crossed safely.

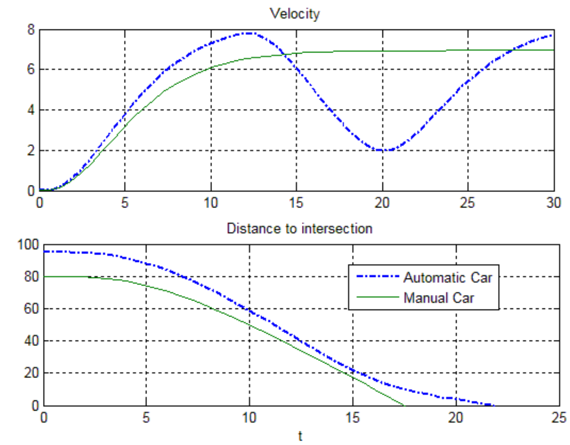

Fig. 8. Simulation result: case one

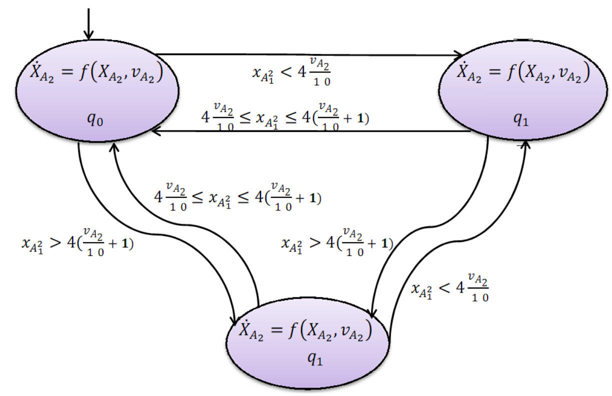

Fig. 9. Hybrid Automaton: Tracking

C. Case Two: Two automatic cars and one manual car

In this case, there are two automatic cars and one manual car which have to cross safely. Apart from that, rear automatic car should always follow another one. Therefore, designing procedure has two part, crossing and tracking.

1) Controller design: The crossing is completely the came as previous section and is applied for both automatic car. In order to design a controller to fix a safe space between two automatic car a hybrid strategy applied. Hybrid Automaton of this controller is shown in Fig. 9.

The same as previous section hybrid automaton will be formulate by (1), where

- $Q = \{q_0, q_1, q_2\}$,
- $X = (x_{A_2}, y_{A_2}, \varphi) \in \mathbb{R}^3$,
- $Init = \{q_0\} \times \{x \in \mathbb{R}^3 : 4\frac{v_{A_2}}{10} \leq x_{A_1^2} \leq 4(\frac{v_{A_2}}{10} + 1)\}$,
- $f = [\ v_{A_2}\cos(\varphi),\quad v_{A_2}\sin(\varphi),\quad v_{A_2}\frac{1}{L}\ ]$,
- $D(q_0) = \{X \in \mathbb{R}^3 : 4\frac{v_{A_2}}{10} \leq x_{A_1^2} \leq 4(\frac{v_{A_2}}{10} + 1)\}$,
- $D(q_1) = \{X \in \mathbb{R}^3 :\ x_{A_1^2} < 4\frac{v_{A_2}}{10}\}$,
- $D(q_2) = \{X \in \mathbb{R}^3 : x_{A_1^2} > 4(\frac{v_{A_2}}{10} + 1)\}$,
- $E = \{(q_0, q_1), (q_1, q_0), (q_0, q_2), (q_2, q_0), (q_1, q_2), (q_2, q_1)\}$,

- $G(q_0,q_1) = G(q_2,q_1) = \{X \in \mathbb{R}^3 : \ x_{A_1^2} < 4\frac{v_{A_2}}{10}\}$,
- $G(q_0,q_2) = G(q_1,q_2) = \{X \in \mathbb{R}^3 : x_{A_1^2} > 4(\frac{v_{A_2}}{10}+1)\}$,
- $G(q_1,q_0) = G(q_2,q_0) = \{X \in \mathbb{R}^3 : 4\frac{v_{A_2}}{10} \leq x_{A_1^2} \leq 4(\frac{v_{A_2}}{10}+1)\}$,
- $R(q_0,q_1,X) = R(q_1,q_0,X) = R(q_0,q_2,X) = R(q_2,q_0,X) = R(q_1,q_2,X) = \{X\}$.

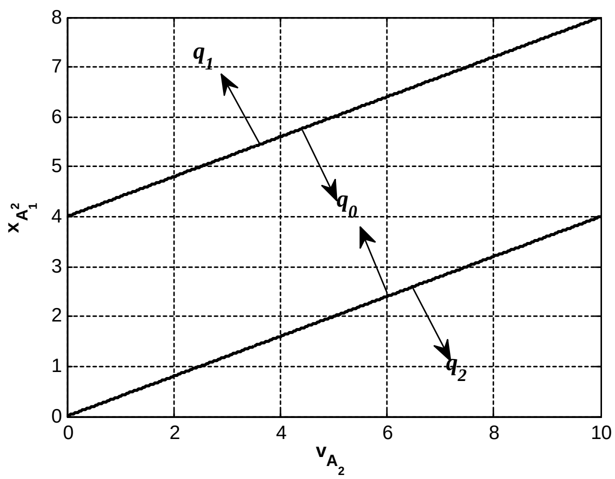

Fig. 10. Domain of each state for case # 2

Domain of each state is shown in Fig. 10. In this case based on the traffic law, the distance between two automatic car is fixed. In this law the safe distance is defined: one car length for every 10 Km/h of speed. This value is considered as minimum distance and the maximum distance is the minimum distance plus length of the car. Length of the car is considered $4\ m$ (based on the length of Citroen C3 which is $3,823mm$). There is no change of velocity if the distance is between the minimum and maximum value i.e. the state q_0 is active. If the distance was less than minimum value the controller will reduce the speed i.e. the state q_1 is active. And If the distance was more than maximum value the controller will increase the speed i.e. the state q_2 is active. Therefore, the reference velocity will be achieved as:

$$v_{dA_2}(t+1) = \begin{cases} v_{dA_2}(t), \text{ if } Q = q_0 \\ 0.9v_{dA_2}(t), \text{ if } Q = q_1 \\ 1.1v_{dA_2}(t), \text{ if } Q = q_2 \end{cases} \quad (5)$$

The same as previous section v_{A_2} can easily obtained using (3).

2) Simulation result: The simulation result is shown in Fig. 11. In this simulation intersection is considered as a segment which its length is 5 meters. Acceleration is limited between -2 and $+2$ based on real data from IAI. The car model is Citroen C3. Furthermore, It assumed there is two intersection i.e. after crossing the first intersection the cars should cross another intersection about 100 meters later. Automatic car No. 1 cross at $t = 11.3\ s$ when the manual car is about 12 meters to cross the intersection number 1 and manual car will cross, at $t = 12.5\ s$ when the automatic car No. 2 is about 7 meters to this intersection. To test the performance of this controller, speed of manual car is reduced at $t = 15\ s$ and it is increased at $t = 20\ s$ in crossing intersection number 2 and automatic cars adopted themselves to this variation by reducing their speed. And, Automatic car No. 1 cross at $t = 23\ s$ when the automatic car No. 2 is about 13 meters to cross the intersection number 2 and automatic car No. 2 will cross, at $t = 24.5\ s$ when the manual car is about 13 meters to this intersection. As it can be seen both automatic car crossed intersections safely. As it is mentioned before, the simulation is also verified the acceleration limit and it is about remain in this margin. Moreover, the distance between two automatic cars is fixed between its maximum and minimum value i.e. $16 < 13.57 < 12$, after crossing. Therefore, the cars cross safely and second automatic car is kept its distance in the allowed values.

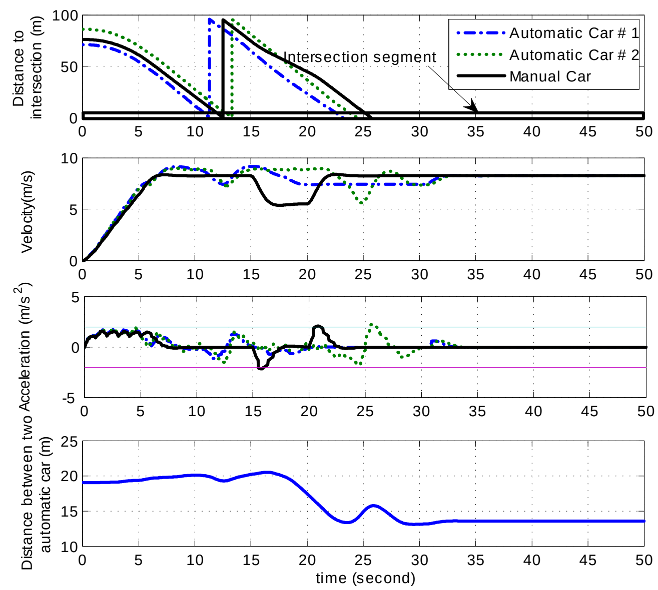

Fig. 11. Simulation result: case two

IV. CONCLUSION

An intelligent crossroad-traversing system aimed at improving traffic flow presented in this paper. The controller is designed by hybrid automaton.The purpose of this paper has been twofold: 1) Safely crossing from intersection and 2) adaptive cruise control. Thus, our system is capable of not only safe crossing the intersection but also keep the safe distance between cars. Simulation result shows the efficiency of the controller. Real experiments are in progress.

ACKNOWLEDGEMENT

This work has been partially supported by research grants TRA2008-06602-C01, TRA2008-06602-C02 (Spanish Min-

istry of science and innovation).

References

[1] Varaiya, P., "Smart cars on smart roads: Problems of control", *IEEE Trans. Autom. Control*, 1993, **38(2)**, 195–207.

[2] Shladover, S. Desoer, C. Hedrick, J. Tomizuka, M. Walrand, J. Zhang, W. McMahon, D. Peng, H., Sheikholeslam,S. and McKeown,N., "Automated vehicle control developments in the PATH program," *IEEE Trans. Veh. Technol.*, 1991, **40(1)**, 114–130.

[3] Highway safety solutions for saving lives, *U.S. Dept. of Transp.*, 2007, **1(2)**.

[4] Milanés,V. Pérez, J. Onieva, E. and González, C., "Controller for Urban Intersections Based on Wireless Communications and Fuzzy Logic", *IEEE Trans. Int. Trans. Sys,* In press.

[5] Febbraro, A. D. Giglion, D. and Saccos, N., "Modular Representation of Urban Traffic Systems based on Hybrid Petri Nets", *IEEE Intelligent Transportation Systems Conference Proceedings - Oakland (CA) USA* , 2001.

[6] Febbraro, A. D. Giglion, D. and Saccos, N., "Urban Traffic Control Structure Based on Hybrid Petri Nets", *IEEE Trans. Int. Trans*, 2004, **5(4)**, 224-237.

[7] Febbraro, A. D. Giglion, D., "Traffic-responsive signalling control through a modular/switching model represented v ia DTPN", *Proceedings of the IEEE Intelligent Transportation Systems Conference Toronto, Canada*, 2006.

[8] Zelinski, S. Koo, T. J. Sastry, S., "Hybrid System Design for Formations of Autonomous Vehicles", *Proceedings of the 42nd IEEE Conference on Decision and Control Maui*, Hawaii USA, 2003.

[9] Giridhar, A. and Kumar, P. R., "Scheduling Automated Traffic on a Network of Roads", *IEEE Trans. Veh. Technol.*, 2006, **55(5)**, 1467-1474.

[10] Li, L. and Wang, F-Y, "Cooperative Driving at Blind Crossings Using Intervehicle Communication", *IEEE Trans. Veh. Technol.*, 2006, **55(6)**, 1712-1724.

[11] Naranjo, J. E. González, C. García, R. and de Pedro, T. "ACC+Stop&Go Maneuvers With Throttle and Brake Fuzzy Control", *IEEE Trans. Int. Trans. Sys.,* 2006, **7(2)**, 213-225.

[12] Hiskens, I.A. and Pai, M.A. "Hybrid systems view of power system modelling", *in Proceedings of the IEEE International Symposium on Circuits and Systems, Geneva, Switzerland, 2000.*

[13] Raibert, M.H. "Legged Robots That Balance", MIT Press, Cambridge, MA, 1986.

[14] Goswami, A. Thuilot, B. and Espiau, B. "A study of the passive gait of a compass-like biped robot: symmetry and chaos", Int. J. Robotics Res., 1998, **17(15)**.

[15] Pettersson, S. "Analysis and design of hybrid systems", Ph.D. Thesis, Department of Signals and Systems, Chalmers University of Technology, Goteborg, Sweden, 1999.

[16] Tomlin, C. Pappas, G. and Sastry, S. "Conflict resolution for air traffic management: A study in multiagent hybrid systems", *IEEE Trans. Aut. Cont.*, 1998, **43(4),** 509-521.

[17] Gven, B.A., Gven, L. "Robust steer-by-wire control based on the model regulator", *In: Proc. Joint IEEE Conf. Control Applications and IEEE Conf. Computer Aided Control Systems Design, Glasgow,,* Los Alamitos, 2002.

[18] Suarez, J.I., Vinagre, B.M., Chen, Y.Q., "A Fractional Adaptation Scheme for Lateral Control of an AGV", *In: FDA 2006. Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and its Applications, Porto, Portugal*, 2006.